

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2018

SECOND YEAR (BATCH 2016-19)

MATH FOR IND. CHEMISTRY (General)

Date : 28/05/2018

Time : 11.00 am – 2.00 pm

Paper : IV

Full Marks : 75

[Use a separate Answer Book for Group A & D and another Answer Book for Group (B+C)]

Group-A

Answer any four questions

[4×5]

1. Evaluate $\int_0^1 \frac{dx}{x^3}$ if exists. Show that $\int_0^\infty e^{-4x} x^{\frac{3}{2}} dx = \frac{3\sqrt{\pi}}{128}$. 5
2. Evaluate $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ if exists. 5
3. Evaluate $\iint (x^2 + y^2) dx dy$ over the region in the positive quadrant for which $x + y \leq 1$. 5
4. Evaluate $\iint_R dx dy$ where R is region bounded by $y^2 = x$ and $x^2 + y^2 = 2x$ in the first quadrant. 5
5. Find the intrinsic equation of the Cardioide $r = a(1 - \cos \theta)$, the arc being measured from the cusp. 5
6. Find the area of the surface generated by revolving about the y -axis the part of the astroid $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, that lies in the first quadrant. 5

Group-B

Answer any four questions

[4×5]

7. Solve: $(x^2 D^2 - 3x D + 5)y = x^2 \sin(\log x)$, where $D \equiv \frac{d}{dx}$. 5
8. Solve: $(D^2 - 9)y = e^{3x} \cos x$, where $D \equiv \frac{d}{dx}$. 5
9. Solve: $\frac{d^2 x}{dt^2} + 4t = 0$, satisfying $x = 4$, $\frac{dx}{dt} = 3$ when $t = 0$. 5
10. Solve: $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = 6x^2$. 5
11. Separate $t^3 u_{xx} + x^3 u_{tt} = 0$ into an equation of x and an equation of t . 5
12. Solve the problem $4w = 0$ on the rectangle $0 < x < a$, $0 < y < b$, with boundary conditions, 5
 $w_x(0, y) = w_x(a, y) = 0$,
 $w(x, 0) = f(x)$, $w_y(x, b) = h(x)$

Group-C

Answer any two questions

[2×5]

13. Verify Green's theorem in the plane for $\oint_c (xy + y^2) dx + x^2 dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5

14. Evaluate $\int \int_S F \cdot n \, dS$, where $F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$. 5
15. If $\vec{a}$ and $\vec{b}$ are constant vectors, prove that $\text{div}\{(\vec{r} \times \vec{a}) \times \vec{b}\} = -2(\vec{a} \cdot \vec{b})$. 5

Group-D

Answer any five questions

[5×5]

16. Define the terms 'absolute error' and 'Percentage error' as used in numerical analysis. The number $\frac{1}{3}$ is approximated as 0.33. Find the absolute, relative and percentage error in this approximation. 2+3
17. Establish the following operator relations:
(i) $(1+\Delta)(1-\nabla) \equiv 1$, (ii) $E \equiv e^{hD}$
where, Δ , ∇ and E are the forward, backward and shift operator respectively also D is the first order differential operator $\left(\frac{d}{dx}\right)$ for independent variable x and h is the step length. 3+2
18. Construct the finite difference table using the values $f(0)=1, f(1)=1.5, f(2)=2.2, f(3)=3.1, f(4)=4.3$.
Also find the value of $f(1.2)$. 2+3
19. Given the following table:

$x :$	0	5	10	15	20
$f(x):$	1.0	1.6	3.8	8.2	15.4

- construct the difference table and compute $f(19)$ by Newton's Backward Formula. 2+3
20. Using Lagrange's interpolation formula, construct the interpolation polynomial for the function $y = \sin \pi x$, choosing the points $x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{1}{2}$. 5
21. Evaluate: $\int_0^1 (4x - 3x^2) dx$ by Simpson's $\frac{1}{3}$ Rule, taking 10 subintervals, correct to 3 places of decimals. Compute the absolute error. 4+1
22. Find the location of the real root of $x^3 - 4x - 9 = 0$ and evaluate this real root by bisection method correct upto four significant figures. 1+4
23. Find by Newton-Raphson method a real root of $3x - \cos x - 1 = 0$ in $(0,1)$ correct to four places of decimals. 5

_____ × _____